DEVELOPMENT OF MAGNETOHYDRODYNAMIC BOUNDARY LAYERS ASSOCIATED WITH THE SUDDEN INITIATION OR DECELERATION OF A SUPERSONIC FLOW AT THE BOUNDARY OF A HALF-SPACE

A. B. Vatazhin

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The problem of nonstationary magnetohydrodynamic flow of a viscous fluid in a half-space resulting from the motion of an infinite plate has received much attention. In [1], for example, solutions are presented for the case of isotropic conductivity, while in [2] a solution of the Rayleigh problem is offered for the case of anisotropic conductivity. In these instances the fluid was assumed incompressible and uniform, and the system of equations was found to be linear. In problems involving nonstationary flow of a gas in a transverse magnetic field resulting from the deceleration of a high-velocity gas flow at the boundary of a half-space or the motion of an infinite plate at supersonic speed relative to a stationary gas it becomes necessary to take into account the compressibility of the gas and the temperature dependence of the conductivity. It is then possible to have flows in which the gas becomes electrically conducting and begins to interact with the magnetic field solely as a result of the increase in temperature due to viscous dissipation of energy. The magnetic field, interacting with the conducting gas, exerts an effect on the drag and heat transfer to the surface of the plate. At sufficiently low gas pressures and strong magnetic fields a Hall effect may be observed. The system of equations describing the motion of a compressible gas with variable conductivity is essentially nonlinear. The present article is devoted to a study of such motions.

§ 1. Consider a nonstationary magnetohydrodynamic flow in which all the quantities depend on a single variable y. From the equations of electrodynamics

$$\operatorname{div} \mathbf{H} = 0, \qquad \operatorname{rot} \mathbf{E} = -c^{-1}\frac{\partial \mathbf{H}}{\partial t}, \qquad \operatorname{rot} \mathbf{H} = 4\pi c^{-1}\mathbf{j},$$

and Ohm's law, written in the form

$$\mathbf{j} = \sigma\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{H}\right) - \alpha\mathbf{j}\times\mathbf{H} \qquad \left(\alpha = \frac{e\tau}{mc}\right),$$

we find that

$$H_y = H^* = \text{const}, \tag{1.1}$$

$$j_x = \frac{\sigma}{1+\beta^2}\left[E_x + \frac{1}{c}(vH_z - wH^*) + \beta E_z + \frac{\beta}{c}(uH^* - vH_x)\right], \tag{1.2}$$

$$j_z = \frac{\sigma}{1+\beta^2}\left[E_z + \frac{1}{c}(uH^* - vH_x) - \beta E_x - \frac{\beta}{c}(vH_z - wH^*)\right], \tag{1.3}$$

$$j_y = j_y(t) = \sigma\left[E_y + \frac{1}{c}(wH_x - uH_z)\right] - \alpha(j_zH_x - j_xH_z) \qquad (\beta = \alpha H^*), \tag{1.4}$$

$$\frac{\partial E_z}{\partial y} = -\frac{1}{c}\frac{\partial H_x}{\partial t}, \qquad \frac{\partial E_x}{\partial y} = \frac{1}{c}\frac{\partial H_z}{\partial t}, \qquad \frac{\partial H_z}{\partial y} = \frac{4\pi}{c}j_x, \qquad -\frac{\partial H_x}{\partial y} = \frac{4\pi}{c}j_z. \tag{1.5}$$

Here

$$\mathbf{H} = (H_x, H_y, H_z), \quad \mathbf{E} = (E_x, E_y, E_z), \quad \mathbf{j} = (j_x, j_y, j_z), \quad \mathbf{v} = (v_x, v_y, v_z)$$

are the vectors of the electric and magnetic fields, the electric current density, and the velocity of the medium, e and m are the electron charge and mass, τ is the mean time between collisions involving an electron and other particles, and c is the speed of light in vacuum.

The equations of continuity, motion, and energy have the form*

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial y} = 0 \tag{1.6}$$

*Since ion slip is not taken into account, the viscous stress tensor does not depend on the magnetic field. The degree of ionization is assumed to be small; therefore the additional, anisotropy-conditioned terms in the heat flux vector may be neglected [3].

$$\rho \frac{\partial u}{\partial t} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y} \mu \frac{\partial u}{\partial y} + X, \qquad X = \frac{j_y H_z - j_z H^*}{c} = \frac{1}{4\pi} H^* \frac{\partial H_x}{\partial y}, \tag{1.7}$$

$$\rho \frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial y} (\lambda + 2\mu) \frac{\partial v}{\partial y} + Y, \; Y = \frac{j_z H_x - j_x H_z}{c} = -\frac{1}{8\pi} \frac{\partial}{\partial y} (H_x^2 + H_z^2), \tag{1.8}$$

$$\rho \frac{\partial w}{\partial t} + \rho v \frac{\partial w}{\partial y} = \frac{\partial}{\partial y} \mu \frac{\partial w}{\partial y} + Z, \qquad Z = \frac{j_x H^* - j_y H_x}{c} = \frac{1}{4\pi} H^* \frac{\partial H_z}{\partial y}, \tag{1.9}$$

$$\rho \frac{\partial h}{\partial t} + \rho v \frac{\partial h}{\partial y} = \frac{\partial}{\partial y} k \frac{\partial T}{\partial y} + \frac{\partial p}{\partial t} + v \frac{\partial p}{\partial y} + (\lambda + 2\mu) \left(\frac{\partial v}{\partial y}\right)^2 + \mu \left(\frac{\partial u}{\partial y}\right)^2 + \mu \left(\frac{\partial w}{\partial y}\right)^2 + \frac{1}{\sigma} (j_x^2 + j_y^2 + j_z^2). \tag{1.10}$$

Here ρ is the density of the medium, p is pressure, T temperature, h enthalpy, c_p specific heat at constant pressure, k and μ thermal conductivity and dynamic viscosity, and λ the second bulk viscosity coefficient.

§ 2. Let the velocity of the gas in the half-space $y > 0$ above the surface of the plate $y = 0$ satisfy the initial and boundary conditions

$$\begin{gathered} u = u_\infty = \text{const} \quad \text{at} \quad 0 < y < \infty, \; t = 0, \; y = \infty, \; t \geqslant 0, \\ \mathbf{v} = 0 \quad \text{at} \quad y = 0, \; t \geqslant 0 . \end{gathered} \tag{2.1}$$

Conditions (2. 1) are realized for the motions of a viscous gas in the following cases: a plate moving at $t < 0$ with the velocity of the surrounding uniform gas flow is instantaneously stopped at the instant $t = 0$; a uniform gas flow in the half-space $y > 0$ instantaneously begins to be decelerated at the boundary $y = 0$ at the instant $t = 0$; at the instant $t = 0$ a plate instantaneously acquires a velocity u_∞ relative to a stationary gas. The coordinate system is tied to the plate. The velocity u_∞ is assumed to be much greater than the speed of sound in the gas at infinity.

Let a uniform external magnetic field H^* be applied in the direction of the y axis. The interaction of the gas with the field will be different depending on whether the magnetic field is tied to the plate (the plate "carries" the magnetic field with it) or to the gas flow at infinity. If one draws the analogy with flow past bodies, then the first case corresponds to motion of a vehicle carrying a magnetic system on board, and the second to the motion of a body in an external magnetic field (e. g. , the earth's magnetic field).

The gas temperature at $t = 0$ is assumed to be not so high that the gas is appreciably electrically conducting and interacts with the magnetic field. At $t > 0$, as a result of deceleration, kinetic energy is converted into thermal energy and the gas is heated. The thickness of the layer of effectively decelerated gas increases with time (a "viscous" wave moves along the y axis). At a sufficiently high velocity of the incident flow the temperature in the layer of decelerated gas increases to the point at which the gas becomes electrically conducting and interacts with the magnetic field. Hence the velocity and temperature profiles become different from the ordinary hydrodynamic ones. In the presence of conductivity anisotropy the field leads to the appearance of a velocity w.

Below we shall consider cases where the enthalpy of the gas satisfies the following initial and boundary conditions:

$$\begin{gathered} h = h_\infty \quad \text{at} \quad 0 < y < \infty, \; t = 0; \; y = \infty, \; t \geqslant 0, \\ h = h_w \quad \text{or} \quad (\partial h / \partial y) = 0 \quad \text{at} \quad y = 0, \; t \geqslant 0. \end{gathered} \tag{2.2}$$

In (2. 2) and elsewhere in this article the subscript w will relate to the parameters of the gas at the surface of the plate. The first of the conditions at the wall corresponds to the case of heat transfer from the surface, the second to the case of a thermally insulated surface. Since $j_y(\infty, t) = 0$, in accordance with (1. 4) $j_y \equiv 0$.

In order to estimate the order of thickness of the layer of decelerated gas $\delta(t)$, we will consider the case $H = 0$, $\rho = \text{const}$, $\mu = \text{const}$. From (1. 6)-(1. 10) it follows that $v = w = 0$, while the velocity u satisfies the known equation

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} \qquad \left(\nu = \frac{\mu}{\rho}\right) \tag{2.3}$$

whose solution for boundary conditions (2. 1) is

$$u = u_\infty \operatorname{erf} \eta \qquad \left(\eta = \frac{y}{2\sqrt{\nu t}}, \; \operatorname{erf} \eta = \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-\eta^2} d\eta\right). \tag{2.4}$$

Equations (2. 4) show that $\delta \sim \sqrt{\nu t}$. When the gas is compressible but the electromagnetic force is smaller than or equal in order of magnitude to the viscous force, then

$$\delta \sim \sqrt{\nu^* t} \qquad (\nu^* = \mu^*/\rho^*) \tag{2. 5}$$

In (2. 5) and elsewhere in this article an asterisk superscript denotes characteristic dimensional quantities.

We shall now estimate the order of magnitude of the various terms in the system (1. 1)-(1. 10), assuming that the characteristic time and the characteristic dimension are related by (2. 5).

In the first place, from continuity equation (1. 6) we find that for a compressible $v \sim \nu^*/\delta$.

If the external applied electric fields are not very strong and $\beta < 1$, then the orders of magnitude of the expressions in square brackets in (1. 2) and (1. 3) are respectively determined by the orders of magnitude of the terms $c^{-1}\beta u H^*$ and $c^{-1}uH^*$. From (1. 5) we find

$$|H_x| \sim \frac{H^* R_m}{1+\beta^{*2}}, \quad |H_z| \sim \frac{H^* R_m}{1+\beta^{*2}}\beta^* \quad \left(R_m = \frac{u_\infty \delta}{\nu_m^*}, \ \nu_m^* = \frac{c^2}{4\pi\sigma^*}\right). \tag{2. 6}$$

The components of the electric field vector E_z and E_x can be written in the form $E_i = E_i'(t) + E_i''(y, t)$ $(i = z, x)$. The component E_i' is determined by the conditions of current flow in the external circuit (if it is assumed that there are current-collecting electrodes connected across the external load), but is also induced as a result of the motion of the plate relative to the external magnetic field; the eddy component E_i'' is a consequence of the time dependence of the induced components of the magnetic field H_x and H_z. Using (2. 5), (2. 6) and the estimate for v, from (1. 5) we get

$$|E_z''| \sim \frac{\chi u_\infty}{c} H^*, \quad |E_x''| \sim \frac{\chi\beta^* u_\infty}{c} H^* \quad \left(\chi = \frac{\nu^*}{\nu_m^*(1+\beta^{*2})}\right). \tag{2. 7}$$

On the basis of these same estimates we have

$$\left|\frac{vH_x}{uH^*}\right| \sim \chi, \quad \left|\frac{vH_z}{\beta u H^*}\right| \sim \chi, \quad \left|\frac{\beta v H_z}{uH^*}\right| \sim \chi\beta^{*2}. \tag{2. 8}$$

The velocity w is a consequence of the action of the force $c^{-1}j_x H^*$ directed along the z axis. From (1. 9) it follows that

$$\frac{w}{u_\infty} \sim \frac{\beta^* \varkappa^2}{1+\beta^{*2}} \qquad \left(\varkappa^2 = \frac{\sigma^* H^{*2}\delta^2}{c^2 \mu^*}\right). \tag{2. 9}$$

If the inequality

$$\Pi_1 = (\nu^*/\nu_m^*) \ll 1 \tag{2. 10}$$

holds, then, on the basis of the above estimates, expressions (1. 2) and (1. 3) may be written in the form

$$\begin{aligned} j_x &= \frac{\sigma}{1+\beta^2}\left(E_x' - \frac{w}{c}H^* + \beta E_z' + \frac{\beta u}{c}H^*\right), \\ j_z &= \frac{\sigma}{1+\beta^2}\left(E_z' + \frac{u}{c}H^* - \beta E_x' + \frac{\beta w}{c}H^*\right). \end{aligned} \tag{2. 11}$$

Let us consider the equations of motion. From (1. 8) and the estimate for it follows that

$$\left|\frac{\Delta T}{p_\infty}\right| \sim \frac{\rho^* u_\infty^2}{p_\infty} R^{-2} \qquad \left(T = p + \frac{1}{8\pi}(H_x^2 + H_z^2), \ R = \frac{u_\infty \delta}{\nu^*}\right).$$

Here and henceforth $\Delta\Phi$ denotes the change in Φ across the layer $\delta(t)$. If

$$\Pi_2 = (\rho^* u_\infty^2/p_\infty) R^{-2} \ll 1 \tag{2. 12}$$

then the change in t with respect to the coordinate y is much smaller than the corresponding change in velocity $(|\Delta u/u| \sim 1)$ and enthalpy $(|\Delta h/h| \sim u_\infty^2/2h_\infty \gtrsim 1)$ and with the indicated accuracy T = const everywhere in the flow (the constant is determined from the conditions in the uniform stationary flow at y = ∞). Using estimates (2. 6), we find

$$\left|\frac{\triangle p}{p_\infty}\right| \leqslant \frac{1}{8\pi p_\infty}(|\triangle H_x^2| + |\triangle H_z^2|) \sim \frac{mR_m^2 H^{*2}}{8\pi p_\infty (1+\beta^{*2})^2} \qquad (m = \max(\beta^{*2}, 1)) .$$

If

$$\Pi_3 = (R_m^2 H^{*2}/8\pi p_\infty) \ll 1 \tag{2.13}$$

then the change in pressure along y may be neglected and $p = p_\infty$ everywhere in the flow.

Satisfaction of inequality (2.12) necessarily requires that $R \gg 1$, and for satisfaction of inequality (2.13) with $(H^{*2}/8\pi p_\infty) > 1$ it is necessary that $R_m \ll 1$. From the condition $R \gg 1$ and the estimate for the velocity v it follows that in (1.10)

$$(\lambda + 2\mu)\left(\frac{\partial v}{\partial y}\right)^2 \ll \mu\left(\frac{\partial u}{\partial y}\right)^2 . \tag{2.14}$$

We shall determine to what extent inequalities (2.10), (2.12), and (2.13) are satisfied in practical applications. At the initial instant let the plate be located in a high-velocity flow of cold air. In order to estimate the maximum temperature in the layer $\delta(t)$ we assume that the stagnation enthalpy profile is similar to the velocity profile, while the temperature of the plate is little different from the temperature T_∞ of the flow at infinity. Then the maximum value of the enthalpy in the layer $\delta(t)$ is approximately equal to $h_\infty(1 + 0.125(\gamma - 1)M^2)$, where M is the Mach number of the oncoming flow. As the characteristic temperature T^* we take the temperature corresponding to this enthalpy and $p = p_\infty$, while as the characteristic hydrodynamic parameters of the air entering into estimates (2.10), (2.12) and (2.13) we take the parameters for $p = p_\infty$ and $T = T^*$. Let[1] $T_\infty = 220°$K, $H^* = 5000$ gauss. Consider the cases $p_\infty = 1$ atm, $M = 18.5$, $p_\infty = 1$ atm, $M = 30$, $p_\infty = 0.001$ atm, $M = 24.7$. The parameter Π_1 is equal, respectively, to $2.9 \cdot 10^{-11}$, $6.7 \cdot 10^{-8}$, $3.2 \cdot 10^{-7}$; the parameter R to $2 \cdot 10^5 \sqrt{t}$, $2 \cdot 10^5 \sqrt{t}$, $7.9 \cdot 10^3 \sqrt{t}$; the parameter Π_2 to $8.4 \cdot 10^{-10} t^{-1}$, $1.2 \cdot 10^{-9} t^{-1}$, $8.4 \cdot 10^{-7} t^{-1}$; the parameter R_m to $5.9 \cdot 10^{-6} \sqrt{t}$, $1.3 \cdot 10^{-2} \sqrt{t}$, $2.5 \cdot 10^{-3} \sqrt{t}$; the parameter $\Pi'_3 R_m^{-1}$ to $5.8 \cdot 10^{-6} \sqrt{t}$, $1.3 \cdot 10^{-2} \sqrt{t}$, $2.5 \sqrt{t}$. In making the estimates we put $\delta = \sqrt{\nu^* t}$. The time t in these estimates is measured in seconds. Clearly, the inequality (2.10) is always satisfied. Inequality (2.12) is satisfied except at very small values of t (when $t = 0$ the basic equations have a singularity, in the same way as the equations of a two-dimensional stationary boundary layer have a singularity at the leading edge of the plate). At not too small pressures inequality (2.13) is satisfied over a wide range of variation of t. As the pressure decreases, the range of variation of t over which (2.13) holds is considerably narrowed.

Henceforth we shall assume that inequalities (2.10), (2.12), and (2.13) are satisfied. The simplified system of equations (relations (2.11) and (2.14) hold, $p = p_\infty$ everywhere in the flow) in the nondimensional variables

$$u = u_\infty u^\circ,\ w = u_\infty w^\circ,\ h = h_\infty h^\circ,\ \theta = \alpha^{-1}(h^\circ - 1) + u^{\circ 2} + w^{\circ 2}$$

$$\mu = \mu_\infty \mu^\circ, \quad \rho = \rho_\infty \rho^\circ, \quad \sigma = \sigma^* \sigma^\circ, \quad E_x' = \frac{u_\infty e_x}{c} H^*, \quad E_z' = \frac{u_\infty e_z}{c} H^*$$

has the form

$$\begin{gathered}
\frac{\partial \rho^\circ}{\partial t} + \frac{\partial \rho^\circ v}{\partial y} = 0, \quad \rho^\circ \frac{\partial u^\circ}{\partial t} + \rho^\circ v \frac{\partial u^\circ}{\partial y} = \nu_\infty \frac{\partial}{\partial y} \mu^\circ \frac{\partial u^\circ}{\partial y} - \varepsilon s (e_z - \beta e_x + u^\circ + \beta w^\circ) \\
\rho^\circ \frac{\partial w^\circ}{\partial t} + \rho^\circ v \frac{\partial w^\circ}{\partial y} = \nu_\infty \frac{\partial}{\partial y} \mu^\circ \frac{\partial w^\circ}{\partial y} + \varepsilon s (e_x - w^\circ + \beta e_z + \beta u^\circ), \\
\rho^\circ \frac{\partial \theta}{\partial t} + \rho^\circ v \frac{\partial \theta}{\partial y} = \nu_\infty \frac{\partial}{\partial y} \mu^\circ P^{-1} \frac{\partial \theta}{\partial y} - \nu_\infty \frac{\partial}{\partial y}\left[\mu^\circ P^{-1}(1 - P) \frac{\partial}{\partial y}(u^{\circ 2} + w^{\circ 2})\right] + \\
+ 2\varepsilon s (e_x^2 + e_z^2 - e_x w + e_z u + \beta e_x u + \beta e_z w)
\end{gathered} \tag{2.15}$$

$$\alpha = \frac{u_\infty^2}{2h_\infty}, \qquad P = \frac{c_p \mu}{k}, \qquad \varepsilon = \frac{\sigma^* H^{*2}}{c^2 \rho_\infty}, \qquad s = \frac{\sigma^\circ}{1+\beta^2} . \tag{2.16}$$

Here quantities with a superscript ° and also θ, e_x, e_z are nondimensional. In (2.16) the quantity α is the compressibility parameter (in the case of a perfect gas with constant specific heats $\alpha = 0.5(\gamma - 1)M^2$), P is the Prandtl number, ε a constant with dimensionality $\sec^{-1}$, and s the nondimensional "effective" conductivity. The quantities ρ°, μ°, P, s, β depend only on the enthalpy h.

The boundary conditions for u°, v, w°, θ follow from (2.1)-(2.2)

$$\begin{gathered}
u^\circ = 1, \quad w^\circ = 0, \quad \theta = 1 \quad \text{at} \quad 0 < y < \infty, \quad t = 0, \quad y = \infty, \quad t \geqslant 0, \\
u^\circ = 0, \quad w^\circ = 0, \quad v = 0 \quad \text{at} \quad y = 0, \ t \geqslant 0
\end{gathered} \tag{2.17}$$

[1]In determining the gasdynamic parameters we used the tables of [4] and [5].

$$\theta = \theta_w = \alpha^{-1}(h_w^\circ - 1) \quad \text{or} \quad (\partial\theta / \partial y) = 0 \quad \text{at} \quad y=0,\ t \geqslant 0. \tag{2.17 (cont'd)}$$

If e_x and e_z are known, then system (2. 15) with boundary conditions (2. 17) is closed. Let the magnetic field be fixed relative to the plate. We shall express e_x and e_z in terms of the parameters of two electrical circuits, respectively connecting electrodes in contact with the gas located in the planes $z = \pm 0.5a_z$ and the planes $x = \pm 0.5a_x$. From the Kirchhoff equations for these circuits, on the assumption that their self-induction emf's can be neglected, using (2. 11) we get

$$\begin{gathered}
\Lambda_z = e_z\left(1 + q_z\int_0^\infty s\,dy\right) + q_z\int_0^\infty s(u^\circ + \beta w^\circ)\,dy - q_z e_x\int_0^\infty s\beta\,dy, \\
\Lambda_x = e_x\left(1 + q_x\int_0^\infty s\,dy\right) + q_x\int_0^\infty s(\beta u^\circ - w^\circ)\,dy + q_x e_z\int_0^\infty s\beta\,dy, \\
\Lambda_i = \frac{cE_i^0}{u_\infty a_i H^*}, \qquad q_i = \frac{\sigma^* R_i}{a_i} \quad (i = z,\ x).
\end{gathered} \tag{2.18}$$

Here E^0_i is the external emf acting in the direction i, and R_i is the external resistance.

If $E^0{}_i = 0$, $q_i = 0$ $(i = z,\ x)$, then from (2. 18) $e_i = 0$. In the case of isotropic conductivity, this regime (the short-circuit regime) is amenable to a graphic interpretation if the plate is considered to be the surface of a cylinder of infinite radius; the streamlines will be concentric circles and the electric field will be zero.

If $E^0_i = 0$, we get the generator regime. If $E^0_i \neq 0$, then, depending on the sign and the magnitude of E^0_i and the time t we may get both generator and accelerator regimes. Finally, by suitably varying the parameters of the external circuits in time, we can make the quantities e_z and e_x constant throughout the entire process.

After solving system (2. 15) using (1. 5) and (2. 11), we determine the induced magnetic field. In the Dorodnitsyn variables

$$\zeta = \int_0^y \rho^\circ\,dy, \qquad \tau = t$$

system (2. 15) assumes the form

$$\begin{gathered}
\frac{\partial u^\circ}{\partial\tau} = \nu_\infty\frac{\partial}{\partial\zeta}\psi\frac{\partial u^\circ}{\partial\zeta} - \varepsilon\varphi(e_z - \beta e_x + u^\circ + \beta w^\circ) \qquad (\psi = \rho^\circ\mu^\circ), \\
\frac{\partial w^\circ}{\partial\tau} = \nu_\infty\frac{\partial}{\partial\zeta}\psi\frac{\partial w^\circ}{\partial\zeta} + \varepsilon\varphi(e_x + \beta e_z + u^\circ\beta - w^\circ) \qquad \left(\varphi = \frac{\sigma^\circ}{(1+\beta^2)\rho^\circ}\right), \\
\frac{\partial\theta}{\partial\tau} = \nu_\infty\frac{\partial}{\partial\zeta}P^{-1}\psi\frac{\partial\theta}{\partial\zeta} - \nu_\infty\frac{\partial}{\partial\zeta}\left[P^{-1}(1-P)\psi\frac{\partial}{\partial\zeta}(u^{\circ 2} + w^{\circ 2})\right] + \\
+ 2\varepsilon\varphi(e_x^2 + e_z^2 - e_x w^\circ + e_z u^\circ + \beta e_x u^\circ + \beta e_z w^\circ),
\end{gathered} \tag{2.19}$$

$$\begin{gathered}
u^\circ = 1, \quad w^\circ = 0, \quad \theta = 1 \quad \text{at } 0 < \zeta < \infty, \quad \tau = 0; \quad \zeta = \infty,\ \tau \geqslant 0, \\
u^\circ = 0, \quad w^\circ = 0, \quad \theta = \theta_w \quad \text{or} \quad (\partial\theta / \partial\zeta) = 0 \quad \text{at} \quad \zeta = 0,\ \tau \geqslant 0.
\end{gathered} \tag{2.20}$$

For conversion to the old variables we use the formulas

$$t = \tau, \qquad y = \int_0^\zeta \frac{d\zeta}{\rho^\circ}. \tag{2.21}$$

In § 3 and 4 below we consider cases where the magnetic field is fixed relative to the plate, and in § 5 the case where the plate moves in an external magnetic field.

§ 3. Consider the case $e_x = 0$, $e_z = 0$. We shall also assume that $\psi = 1$, $P = 1$. These latter conditions are widely accepted assumptions in the theory of a compressible boundary layer. From (2. 19) we get the system

$$\frac{\partial u^\circ}{\partial\tau} = \nu_\infty\frac{\partial^2 u^\circ}{\partial\zeta^2} - \varepsilon\varphi(u^\circ + \beta w^\circ), \qquad \frac{\partial w^\circ}{\partial\tau} = \nu_\infty\frac{\partial^2 w^\circ}{\partial\zeta^2} + \varepsilon\varphi(u^\circ\beta - w^\circ), \tag{3.1}$$

$$\frac{\partial\theta}{\partial\tau} = \nu_\infty\frac{\partial^2\theta}{\partial\zeta^2}. \tag{3.2}$$

Equation (3. 2) has the same form as (2. 3) and its solution is

$$\theta = \theta_w + (1 - \theta_w)\,\mathrm{erf}\,\eta \quad \text{or} \quad \theta \equiv 1 \qquad (\eta = \zeta / 2\sqrt{\nu_\infty \tau}) .$$

The solution obtained allows us to express the enthalpy of the gas in terms of its velocity

$$h^\circ = 1 - (1 - h_w^\circ)\,\mathrm{Erf}\,\eta + \alpha\,(\mathrm{erf}\,\eta - u^{\circ 2} - w^{\circ 2})$$

or

$$h^\circ = 1 + \alpha\,(1 - u^{\circ 2} - w^{\circ 2}) \quad (\mathrm{Erf}\,\eta = 1 - \mathrm{erf}\,\eta) . \tag{3. 3}$$

In accordance with (3. 3), the enthalpy of the gas at a thermally insulated surface is equal to the stagnation enthalpy of the flow at infinity, while in the case of heat transfer from the surface the heat flux to the wall

$$S = \frac{\nu_\infty}{u_\infty}(\pi t \nu_\infty)^{-1/2} \qquad \left(S = \frac{2\,(k\partial T / \partial y)_w}{\rho_\infty u_\infty (2h_\infty + u_\infty{}^2 - 2h_w)}\right), \tag{3. 4}$$

where S is the Stanton number. Clearly, the heat flux to the surface does not depend on the magnetic field.

Assuming that $\varepsilon\tau$ is a small quantity, we will find the solution of (3. 1), (3. 2) in the form of series in powers of $\varepsilon\tau$ with coefficients depending on the variable η:

$$\begin{aligned} u^\circ &= u_0(\eta) + \varepsilon\tau u_1(\eta) + \varepsilon^2\tau^2 u_2(\eta) + \ldots \\ w^\circ &= \varepsilon\tau w_1(\eta) + \varepsilon^2\tau^2 w_2(\eta) + \ldots \end{aligned} \tag{3. 5}$$

Here and henceforth quantities with the subscript ν ($\nu = 0, 1, 2, \ldots$) are dimensionless and correspond to the ν-th approximation. Substituting expansions (3. 5) into (3. 3), we get an expansion for h° whose first coefficient has the form

$$h_0 = 1 - (1 - h_w{}^\circ)\,\mathrm{Erf}\,\eta + \alpha\,(\mathrm{erf}\,\eta - u_0{}^2) \quad \text{or} \quad h_0 = 1 + \alpha\,(1 - u_0{}^2), \tag{3. 6}$$

while the subsequent coefficients h_i ($i = 1, 2, \ldots$), identical for both relations (3. 3), are equal to

$$h_1 = -2\alpha u_0 u_1, \qquad h_2 = -\alpha\,(u_1{}^2 + 2u_0 u_2 + w_1{}^2), \ldots \tag{3. 7}$$

The series expansion of the function $\varphi(h^\circ)$ is found using the series expansion of the quantity h°:

$$\begin{aligned} &\varphi = \sum_{k=0}^{\infty} (\varepsilon\tau)^k \varphi_k, \quad \varphi_0 = \varphi(h_0), \quad \varphi_1 = h_1 \left(\frac{d\varphi}{dh^\circ}\right)_{h^\circ = h_0} \\ &\varphi_2 = h_2 \left(\frac{d\varphi}{dh^\circ}\right)_{h^\circ = h_0} + \frac{h_1{}^2}{2}\left(\frac{d^2\varphi}{dh^{\circ 2}}\right)_{h^\circ = h_0}, \ldots \end{aligned} \tag{3. 8}$$

The coefficients of the expansion of the function $\beta(h^\circ)$ are given by the same formulas (3. 8), if β is substituted for φ.

Substituting (3. 5) and (3. 8) into (3. 1) and equating the coefficients of like powers of $\varepsilon\tau$, we get the following linear ordinary differential equations:

$$u_0'' + 2\eta u_0' = 0, \qquad u_0(0) = 0, \qquad u_0(\infty) = 1, \tag{3. 9}$$

$$u_1'' + 2\eta u_1' - 4u_1 = 4\varphi_0 u_0, \qquad u_1(0) = 0, \qquad u_1(\infty) = 0, \tag{3. 10}$$

$$w_1'' + 2\eta w_1' - 4w_1 = -4\varphi_0 u_0 \beta_0, \qquad w_1(0) = 0, \qquad w_1(\infty) = 0, \tag{3. 11}$$

$$u_2'' + 2\eta u_2' - 8u_2 = 4(\varphi_1 u_0 + \varphi_0 u_1 + \varphi_0 \beta_0 w_1), \; u_2(0) = 0, \; u_2(\infty) = 0, \tag{3. 12}$$

$$w_2'' + 2\eta w_2' - 8w_2 = -4\,(\varphi_0 u_1 \beta_0 + \varphi_0 u_0 \beta_1 - \varphi_0 w_1 + u_0 \beta_0 \varphi_1),$$

$$w_2(0) = 0, \quad w_2(\infty) = 0 . \tag{3. 13}$$

Here the prime denotes differentiation with respect to the variable η. The above and subsequent equations are solved successively and integrated in quadratures [6]. We shall confine ourselves to a consideration of the zero and first approximations. Consider the equation

$$\Phi'' + 2\eta\Phi' - 4\Phi = 2f(\eta), \qquad |f(\eta)|_{(\eta\to\infty)} < A\eta^{-p} \qquad (p > 2). \tag{3. 14}$$

A number of problems in the theory of an unsteady hydrodynamic boundary layer reduce to the solution of an equation of this type [7].

If $\Phi(\infty) = 0$, its solution takes the form

$$\Phi(\eta) = \Phi(0)\exp(-\eta^2)\,G(\eta) + L[\eta, f(\eta)] \qquad (3.15)$$

$$L[\eta, f(\eta)] = -0.5\sqrt{\pi}\,(1+2\eta^2)\Big\{C\,\mathrm{erf}\,\eta + \int_0^\eta f(\eta)\,[(1+2\eta^2)\,\mathrm{erf}\,\eta \times$$

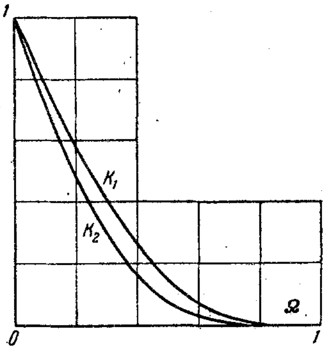
$$\times \exp\eta^2 + 2\pi^{-1/2}\eta]\,d\eta - \mathrm{erf}\,\eta\int_0^\eta f(\eta)\,(1+2\eta^2)\exp\eta^2 d\eta\Big\} -$$

$$- C\eta\exp(-\eta^2) + \eta\exp(-\eta^2)\int_0^\eta f(\eta)(1+2\eta^2)\exp\eta^2 d\eta\,,$$

$$C = \int_0^\infty f(\eta)\,G(\eta)\,d\eta, \qquad \Phi'(0) = -\frac{4\Phi(0)}{\sqrt{\pi}} - 2C\,,$$

$$G(\eta) = (1+2\eta^2)\,\mathrm{Erf}\,\eta\exp\eta^2 - 2\pi^{-1/2}\eta \qquad (G'' - 2\eta G' - 6G = 0). \qquad (3.16)$$

In (3.15) the function $L[\eta, f(\eta)]$ is a solution of (3.14) for $\Phi(0) = 0$, $\Phi(\infty) = 0$, and the function $\Phi(0)\exp(-\eta^2)G(\eta)$ is a solution of the corresponding homogeneous equation for $\Phi(\infty) = 0$. It is easy to show that



$$G(\eta) > 0, \qquad G'(\eta) < 0, \qquad G''(\eta) > 0 \qquad \text{for } 0 < \eta < \infty\,,$$

$$G(0) = 1, \quad G'(0) = -4\pi^{-1/2}, \quad G''(0) = 6, \qquad (3.17)$$

$$G(\infty) = 0, \quad G'(\infty) = 0, \qquad G''(\infty) = 0\,.$$

Graphs of the functions $G(\eta) = K_1(\Omega)$ and $[G''(\eta)/6] = K_2(\Omega)$, where $\Omega = \eta(1+\eta)^{-1}$, are presented in the figure. Further, we have

$$L[\eta, k_1f_1 + k_2f_2] = k_1L[\eta, f_1] + k_2L[\eta, f_2] \quad (k_1, k_2 = \text{const}), \qquad (3.18)$$

$$L[\eta, f(\eta)] \leqslant 0, \qquad \text{if} \qquad f(\eta) \geqslant 0\,. \qquad (3.19)$$

Inequality (3.19) was proved in [8] where one of the problems was reduced to an investigation of Eq. (3.14).

The solutions of Eqs. (3.10) and (3.11) can now be represented in the form

$$u_0 = \mathrm{erf}\,\eta, \qquad u_1 = 2L[\eta, \varphi_0 u_0], \qquad w_1 = -2L[\eta, \varphi_0 u_0 \beta_0] \qquad (3.20)$$

Since $\varphi_0 u_0 \geqslant 0$, $\varphi_0 u_0 \beta_0 \geqslant 0$, on the basis of (3.19) $u_1 \leqslant 0$, $w_1 \geqslant 0$. Thus, the velocity component conditioned by the Hall effect is nowhere directed in the negative direction of the z axis.

We shall show that for fixed values of y and t ($0<y<\infty$, $0<t<\infty$) the velocity u for $H^* \neq 0$ is smaller than for $H^* = 0$. From (2.21) and (3.7) we find

$$\frac{y}{2\sqrt{\nu_\infty t}} = \int_0^\eta \frac{d\eta}{\rho_0} - 2\alpha\varepsilon t\int_0^\eta \left[\frac{d(1/\rho^\circ)}{dh^\circ}\right]_{h^\circ = h_0} u_0 u_1\,d\eta + \ldots$$

Since the derivative in the square brackets in the second integral is positive, $u_0 \geq 0$, $u_1 \leq 0$, denoting by η_{**} and η^{**} values of η corresponding to the fixed quantity $y/2\sqrt{\nu_\infty t}$ for $\varepsilon\tau \neq 0$ and $\varepsilon\tau = 0$, we find that $\eta_{**} \leq \eta^{**}$. Using the fact that $u_0' \geq 0$, we obtain the required inequality

$$u_0(\eta_{**}) + \varepsilon t u_1(\eta_{**}) \leqslant u_0(\eta_{**}) \leqslant u_0(\eta^{**})$$

Let us determine the integral characteristics. The friction drag c_f and the total drag c_d are found using (3.16)

$$c_f = \frac{2\nu_\infty}{u_\infty}(\pi t\nu_\infty)^{-1/2}\left(1 - 2\sqrt{\pi}\,\varepsilon t\int_0^\infty \varphi_0 u_0 G\,d\eta + \ldots\right) \quad \left(c_f = \frac{2\mu_w(\partial u/\partial y)_w}{\rho_\infty u_\infty^2}\right),$$
$$c_d = \frac{2\nu_\infty}{u_\infty}(\pi t\nu_\infty)^{-1/2}\left(1 + 2\sqrt{\pi}\,\varepsilon t\int_0^\infty \varphi_0 u_0\,(1-G)\,d\eta + \ldots\right). \tag{3.21}$$

$$\left(c_d = c_f + \frac{2H^*}{c\rho_\infty u_\infty^2}\int_0^\infty j_z\,dy\right). \tag{3.22}$$

Since the quantities φ_0, u_0, $1 - G$ are nonnegative, from (3.21) and (3.22) it follows that the magnetic field leads to a decrease in the friction drag and to an increase in the total drag. We assume that the enthalpy dependences of δ, $1 + \beta^2$, and ρ for $h > h_\infty$ are respectively approximated by power relations with exponents n_δ, n_β, and n_ρ:

$$\sigma = \sigma^*\left(\frac{h}{h_\sigma}\right)^{n_\sigma},$$
$$1 + \beta^2 = (1 + \beta^{*2})\left(\frac{h}{h_\beta}\right)^{n_\beta},$$
$$\rho = \rho^*\left(\frac{h}{h_\rho}\right)^{n_\rho}.$$

n	i_1	i_2	i_3	i_4
1	0.06243	0.1990	0.1857	0.4633
3	0.00263	0.1093	$5.69\cdot10^{-3}$	0.1359
4	$5.84\cdot10^{-4}$	0.0909	$1.20\cdot10^{-3}$	0.0959
5	$1.33\cdot10^{-4}$	0.0781	$2.63\cdot10^{-4}$	0.0727
7	$7.13\cdot10^{-6}$	0.0615	$1.36\ 10^{-5}$	0.0474
10	$0.95\cdot10^{-7}$	0.0472	$1.74\cdot10^{-7}$	0.0298

Here δ^*, β^*, ρ^* are certain characteristic values of δ, β, and ρ, computed respectively from the enthalpies h_δ, h_β, and h_ρ. Then

$$\varphi_0 = \frac{\rho_\infty}{\rho^*}\left(\frac{h_\infty}{h_\sigma}\right)^{n_\sigma}\left(\frac{h_\infty}{h_\beta}\right)^{-n_\beta}\left(\frac{h_\infty}{h_\rho}\right)^{-n_\rho}(1 + \beta^{*2})^{-1}h_0^n \qquad (n = n_\sigma - n_\beta - n_\rho) \tag{3.23}$$

Since the variation of φ is basically determined by the change in conductivity (n close to n_δ), while for $h_0 \approx 1$ the conductivity is practically equal to zero, in (3.23) we may assume that

$$h_0 = \alpha \operatorname{erf}\eta \operatorname{Erf}\eta \quad \text{or} \quad h_0 = \alpha\,(1 - \operatorname{erf}^2\eta).$$

The results of evaluation of the integrals in (3.21) and (3.22),

$$i_1 = \sqrt{\pi}\int_0^\infty (\operatorname{erf}\eta \operatorname{Erf}\eta)^n \operatorname{erf}\eta\, G(\eta)\,d\eta, \quad i_2 = \sqrt{\pi}\int_0^\infty \operatorname{erf}\eta\,(1 - \operatorname{erf}^2\eta)^n G(\eta)\,d\eta,$$

$$i_3 = \sqrt{\pi}\int_0^\infty (\operatorname{erf}\eta \operatorname{Erf}\eta)^n \operatorname{erf}\eta\,(1 - G)\,d\eta,$$

$$i_4 = \sqrt{\pi}\int_0^\infty \operatorname{erf}\eta\,(1 - \operatorname{erf}^2\eta)^n (1 - G)\,d\eta,$$

are presented in the table.

§ 4. Consider the case $\psi = 1$, $P = \text{const} \neq 1$, $e_z = e_x = 0$. The corresponding system of equations consists of (3.1) and the equation

$$\frac{\partial\theta}{\partial\tau} = \nu_\infty P^{-1}\frac{\partial^2\theta}{\partial\zeta^2} - \nu_\infty P^{-1}(1 - P)\frac{\partial^2}{\partial\zeta^2}(u^{\circ 2} + w^{\circ 2}). \tag{4.1}$$

The solution of this system will be found in the form of series (3.5) and the series

$$\theta = \theta_0 + \varepsilon\tau\theta_1 + \varepsilon^2\tau^2\theta_2 + \ldots \tag{4.2}$$

Using these series, we determine the series expansion of the enthalpy h°

$$h^\circ = h_0 + \varepsilon\tau h_1 + \varepsilon^2\tau^2 h_2 + \ldots \qquad h_0 = 1 + \alpha\,(\theta_0 - u_0^2), \tag{4.3}$$

$$h_1 = \alpha(\theta_1 - 2u_0u_1), \qquad h_2 = \alpha(\theta_2 - 2u_0u_2 - u_1^2 - w_1^2). \qquad (4.3)\ \text{(cont'd)}$$

The functions $\varphi(h^\circ)$ and $\beta(h^\circ)$ are expanded in series of type (3.8) in which h_0, h_1 and h_2 are expressed by formulas (4.3). Substituting (3.5) and (4.2) into (3.1) and (4.1), we get equations (3.9)-(3.13) and the equation

$$\theta_{0\lambda}'' + 2\lambda\theta_{0\lambda}' = (1-P)(u_0^2)_\lambda'' \quad \lambda = \sqrt{P}\eta \quad \theta_0(0) = \alpha^{-1}(h_w^\circ - 1) \text{ or } \theta_{0\lambda}'(0) = 0,\ \theta_0(\infty) = 1, \quad (4.4)$$

$$\theta_{1\lambda}'' + 2\lambda\theta_{1\lambda}' - 4\theta_1 = 2(1-P)(u_0u_1)_\lambda'' \quad \theta_1(0) = 0 \quad \text{or} \quad \theta_{1\lambda}'(0) = 0,\ \theta_1(\infty) = 0, \quad (4.5)$$

$$\theta_{2\lambda}'' + 2\lambda\theta_{2\lambda}' - 8\theta_2 = (1-P)(u_1^2 + 2u_0u_2 + w_1^2)_\lambda'' \quad \theta_2(0) = 0 \text{ or } \theta_{2\lambda}'(0) = 0, \quad \theta_2(\infty) = 0. \quad (4.6)$$

In (4.4)-(4.6) and below the subscript λ indicates that differentiation is performed with respect to the variable λ. If there is no subscript, then differentiation is performed with respect to the variable η. Each equation in θ_i for known u_k $(k = 0, \ldots, i)$ and w_k $(k = 1, \ldots, i-1)$ and each equation in u_i and w_i for known θ_k, u_k, w_k $(k = 0, \ldots, i-1)$ is integrated in quadratures. Therefore the system of equations may be successively integrated in quadratures. For the zeroth and first approximations we have:

$$\begin{gathered} u_0 = \operatorname{erf}\eta, \quad \theta_0 = \alpha^{-1}(h_w^\circ - 1) + 0.5\sqrt{\pi A}\operatorname{erf}\lambda + (1-P)\,r(\lambda) \\ \text{or} \quad \theta_0 = 1 - (1-P)\,g\sqrt{\pi} + (1-P)\,r(\lambda), \\ A = 2\pi^{-1/2}[\alpha^{-1}(1 + \alpha - h_w^\circ) - g\sqrt{\pi}(1-P)], \\ g = \int_0^\infty u_0^2 G(\lambda)\,d\lambda, \quad r(\lambda) = \int_0^\lambda \exp(-\lambda^2)\left(\int_0^\lambda (u_0^2)_\lambda'' \exp\lambda^2 d\lambda\right)d\lambda, \end{gathered} \qquad (4.7)$$

$$\begin{gathered} u_1 = 2L[\eta, \varphi_0 u_0], \quad w_1 = -2L[\eta, \varphi_0 u_0 \beta_0], \quad \theta_1 = (1-P)\,L[\lambda, (u_0u_1)_\lambda''] \\ \text{or} \quad \theta_1 = -0.5d\sqrt{\pi}(1-P)\exp(-\lambda^2)\,G(\lambda) + (1-P)\,L[\lambda, (u_0u_1)_\lambda''], \\ d = \int_0^\infty u_0u_1G_\lambda''(\lambda)\,d\lambda. \end{gathered} \qquad (4.8)$$

The coefficients c_f and c_d are determined from (3.21), (3.22), in which $\varphi_0 = \varphi(h_0)$, where $h_0 = 1 + \alpha(\theta - u_0^2)$. The Stanton number in the case of flow over a surface with heat transfer and the enthalpy of the gas at a thermally insulated surface are found using (4.7) and (4.8):

$$S = \frac{v_\infty}{u_\infty}(\pi t v_\infty P)^{-1/2}\left[1 - \frac{\alpha g\sqrt{\pi}(1-P)}{1 + \alpha - h_w^\circ} - \frac{\varepsilon t \alpha d\sqrt{\pi}(1-P)}{1 + \alpha - h_w^\circ} + \ldots\right], \qquad (4.9)$$

$$h_w^\circ = 1 + \alpha[1 - g\sqrt{\pi}(1-P)] - 0.5\varepsilon t\alpha d\sqrt{\pi}(1-P). \qquad (4.10)$$

In deriving (4.8)-(4.10) we used the equality

$$d = \int_0^\infty (u_0u_1)_\lambda'' G(\lambda)\,d\lambda = \int_0^\infty u_0u_1G_\lambda''(\lambda)\,d\lambda.$$

Since $u_0 \le 0$, $u_1 \le 0$, $G'' \ge 0$, we have $d \le 0$. Thus, if $P < 1$, the magnetic field leads to an increase in heat flux, while $P > 1$ it leads to a decrease. Similarly, when a magnetic field is applied the enthalpy of the gas at a thermally insulated surface decreases if $P > 1$ and increases if $P < 1$.

It is of interest to compare the results obtained in § 3, 4 with the results of [9, 5] in which studies were made of the stationary MHD boundary layer on a flat plate to which a transverse magnetic field was applied. In [9], by numerical integration of the boundary layer equations for the case $P = 1$, constant ρ, μ, and k, and an exponential dependence of conductivity on temperature Rossow found that a magnetic field leads to a decrease in friction drag and heat flux and to an increase in total drag. Upon development of a boundary layer, in accordance with § 3, when P = 1 the magnetic field acts on the friction and total drag in the same sense, but does not affect the heat flux to the surface. In [5] Bush numerically integrated the equations of a compressible boundary layer on a flat plate to which he applied a transverse magnetic field decreasing from the leading edge $1/\sqrt{x}$. It was assumed that the plate was located in a flow of air and that P = 0.70, while allowance was made for the variation of ρ, μ, and δ. Calculations showed that a magnetic field leads to a decrease in friction and heat flux. Upon development of a boundary layer, in accordance with the results of § 4, when P = 0.70 < 1 the magnetic field reduces friction but increases the heat flux.

§ 5. Let the magnetic field be fixed relative to the oncoming flow (plate moving in an external magnetic field). In this case $e_z = -1$. We put $e_x = 0$, $\psi = 1$, P = 1; system (2.19) assumes the form

$$\frac{\partial u^\circ}{\partial \tau} = \nu_\infty \frac{\partial^2 u^\circ}{\partial \zeta^2} + \varepsilon\varphi(1 - u^\circ - \beta w^\circ), \quad \frac{\partial \theta}{\partial \tau} = \nu_\infty \frac{\partial^2 \theta}{\partial \zeta^2} + 2\varepsilon\varphi(1 - u^\circ - \beta w^\circ), \quad \frac{\partial w^\circ}{\partial \tau} = \nu_\infty \frac{\partial^2 w^\circ}{\partial \zeta^2} - \varepsilon\varphi[\beta(1 - u^\circ) + w^\circ]. \tag{5.1}$$

Note that in this case, in order that the flow at infinity be uniform, it is not necessary to require that $\varphi \to 0$ as $y \to \infty$. Therefore system (5.1) with boundary conditions (2.20) can be used not only to describe the motion of a plate in an initially nonconducting gas, when the conductivity in the boundary layer increases as a result of heating due to friction, but also to describe the motion of a plate in a fluid of constant conductivity.

We shall seek the solution of system (5.1) in the form of series (3.5) and (4.2). Substituting these series into (5.1), for the zero approximation (u_0 and θ_0) we get Eqs. (3.9) and (4.4) whose solutions are given by (4.7) (in (4.4) and (4.7) we must put P = 1, $\eta = \lambda$), and for the first approximation the equations

$$u_1'' + 2\eta u_1' - 4u_1 = -4\varphi_0 \operatorname{Erf}\eta, \qquad u_1(0) = 0, \qquad u_1(\infty) = 0 \tag{5.2}$$

$$w_1'' + 2\eta w_1' - 4w_1 = 4\varphi_0\beta_0 \operatorname{Erf}\eta, \qquad w_1(0) = 0, \qquad w_1(\infty) = 0, \tag{5.3}$$

$$\theta_1'' + 2\eta\theta_1' - 4\theta_1 = -8\varphi_0 \operatorname{Erf}\eta, \qquad \theta_1(0) = 0 \quad \text{or} \quad \theta_1'(0) = 0, \qquad \theta_1(\infty) = 0. \tag{5.4}$$

We recall that here $\varphi_0 = \varphi(h_0)$, $\beta_0 = \beta(h_0)$, where $h_0 = 1 + \alpha(\theta_0 - u_0^2)$. The solution of these equations has the form

$$u_1 = -2L[\eta, \varphi_0 \operatorname{Erf}\eta], \qquad w_1 = 2L[\eta, \varphi_0\beta_0 \operatorname{Erf}\eta],$$

$$\theta_1 = -4L[\eta, \varphi_0 \operatorname{Erf}\eta]$$

or

$$\theta_1 = 0.5m\sqrt{\pi}\exp(-\eta^2)G(\eta) - 4L[\eta, \varphi_0 \operatorname{Erf}\eta] \tag{5.5}$$

$$\left(m = 4\int_0^\infty G(\eta)\varphi_0 \operatorname{Erf}\eta\, d\eta\right).$$

From (5.5), using (3.19), we find that $u_1 \geqslant 0$, $w_1 \leqslant 0$. The friction drag, Stanton number, and the enthalpy of the gas at an insulated wall are equal to

$$c_f = \frac{2\nu_\infty}{u_\infty}(\pi t\nu_\infty)^{-1/2}(1 + 0.5\sqrt{\pi}m\varepsilon t + \ldots) \tag{5.6}$$

$$S = \frac{\nu_\infty}{u_\infty}(\pi t\nu_\infty)^{-1/2}\left(1 + \frac{\alpha m\sqrt{\pi}\varepsilon t}{1 + \alpha - h_w^\circ} + \ldots\right) \tag{5.7}$$

$$h_w^\circ = 1 + \alpha + 0.5m\alpha\sqrt{\pi}\varepsilon t + \ldots \tag{5.8}$$

Since $G \geqslant 0$, $\varphi_0 \geqslant 0$, $\operatorname{Erf}\eta \geqslant 0$, from (5.6) and (5.7) it follows that a magnetic field leads to an increase in friction drag and heat flux. Note that in the case of a two-dimensional stationary boundary layer on a semi-infinite plate moving in an external magnetic field, the friction drag and heat flux due to the action of the magnetic field at P = 1 vary in the same sense [9, 10].

When φ_0 = const (e. g., δ, ρ, and β are constant), we have

$$c_f = \frac{2}{u_\infty}\left(\frac{\nu_\infty}{\pi t}\right)^{1/2}(1 + \varepsilon t\varphi_0 + \ldots), \qquad u_1 = \varphi_0\{\mathrm{Erf}\,\eta - \exp(-\eta^2)\,G(\eta)\},$$

$$S = \frac{\nu_\infty}{u_\infty}(\pi t\nu_\infty)^{-1/2}\left(1 + \frac{2\alpha\varepsilon t\varphi_0}{1 + \alpha - h_w^\circ} + \ldots\right) \quad (m = 2\pi^{-1/2}\varphi_0).$$

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